

# Bosonization and IBM \*

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We derive a boson Hamiltonian from a Nuclear Hamiltonian whose potential is expanded in pairing multipoles and determine the fermion-boson mapping of operators. We use a new method of bosonization based on the evaluation of the partition function restricted to the bosonic composites of interest. By rewriting the partition function so obtained in functional form we get the euclidean action of the composite bosons from which we can derive the Hamiltonian. Such a procedure respects all the fermion symmetries.

## I. INTRODUCTION

The IBM of Arima and Iachello [1], is most successful in describing the low energy nuclear excitations. The bosons of this model are understood as virtual pairs of nucleons, analogous to the Cooper pairs of superconductivity [2]. But no general procedure to reformulate the nuclear theory in terms of the effective bosonic degrees of freedom has been found.

The first attempt in this direction has been performed, as far as we know, by Beliaev and Zelevinsky [3]. But this work makes use of the Bogoliubov transformation which violates nucleon number conservation. Moreover the bosonization is achieved only within a perturbation scheme.

The first work which relates the IBM to a nucleon Hamiltonian is due to Otsuka, Arima and Iachello [4]. These authors got exact results for the pairing interaction in a single  $j$ -shell. Their result was somewhat generalized [5], but a full solution of the problem has not yet been achieved.

There are several recipes for bosonization [6], mostly based on the idea of mapping a fermion model space into a boson space. This requires a truncation of the nucleon space whose effect is in general not easy to control.

In order to avoid the limitations of previous works we try a different approach where we do not assume any property of the composites, other than their dominance at low energy. In particular their structure will be determined only at the end of the calculation. The problem of truncation of the nucleon space will then be traded by the problem of decoupling some bosons from the others, but in a setting where one can hopefully have a better control.

To implement Boson Dominance we perform a functional evaluation of the partition function restricted to boson composites. In this way we get the euclidean action of these composites and their coupling to external fields in closed form. All the fermion symmetries, in particular fermion number conservation, are respected. The bosonization is therefore achieved in the path integral formalism, and all physical quantities can be evaluated by standard methods. The first step, necessary also in the derivation of the Hamiltonian, is to find the minimum of the action at constant fields. Depending on the solution, one has spherical or deformed nuclei. In the latter case rotational excitations appear as Goldstone modes associated to the spontaneous breaking of rotational symmetry. The notion of spontaneous symmetry breaking survives in fact with a precise definition also in finite systems [7]. We want to emphasize that the closed form of the action opens the way to numerical simulations of fermionic systems in terms of bosonic variables, avoiding the "sign problem".

To compare with the IBM we can either write the path integral of the latter, or derive the Hamiltonian corresponding to our action. We will make here the second choice. But to derive the Hamiltonian we must perform an expansion in the inverse of the shell degeneracy.

Bosonization appears in several many-fermion systems and relativistic field theories. The effective bosons fall into two categories, depending on their fermion number. The Cooper pairs of the BCS model of superconductivity, of the IBM of Nuclear Physics, of the Hubbard model of high  $T_c$  superconductivity [8] and of color superconductivity in QCD have fermion number 2. Similar composite bosons with fermion number zero appear as phonons, spin waves and chiral mesons in QCD. They can be included in the present formalism by replacing in the composites one fermion operator by an antifermion (hole) one. Indeed the approach we are going to present can be applied, as far as we can see without any conceptual difficulty, in all the above cases, as it has been argued in a brief report of the method [9].

A different approach to bosonization which also avoids any mapping is based on the Hubbard-Stratonovich transformation. The latter renders quadratic the fermionic interaction by introducing bosonic auxiliary fields which in the

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end become the physical fields. The typical resulting structure is that of chiral theories [10]. In such an approach an energy scale emerges naturally, and only excitations of lower energy can be described by the auxiliary fields [7]. At present the relation with the present approach has not been fully clarified.

The paper is organized in the following way. In Sec. 2 we define coherent states of composites and their properties. In Sec. 3 we derive the path integral for the composites and find the effective bosonic action. We restrict ourselves for simplicity to a nuclear interaction given as a sum of pairing multipoles, but more general forces can easily be included and will be discussed in future works. The effective action we derive is, apart from the above limitation, general. In Sec. 4 we restrict ourselves to a single  $j$ -shell with pairing multipoles and in Sec. 5 we determine the corresponding Hamiltonian. In Sec. 6 we report our conclusions.

## II. COHERENT STATES OF COMPOSITES

Composites of fermion number 2 are defined in terms of the fermion creation operators  $\hat{c}^\dagger$

$$\hat{b}_J^\dagger = \frac{1}{2} \hat{c}^\dagger B_J^\dagger \hat{c}^\dagger = \frac{1}{2} \sum_{m_1, m_2} \hat{c}_{m_1}^\dagger (B_J^\dagger)_{m_1, m_2} \hat{c}_{m_2}^\dagger. \quad (1)$$

In the above equation the  $m$ 's represent all the fermion intrinsic quantum numbers and position coordinates and  $J$  the corresponding labels of the composites. Composites of fermion number zero can be obtained by replacing one of the fermion operators by an antifermion one. The structure matrices  $B_J$  have dimension  $2\Omega$  independent of  $J$ . Their form is determined by the fermion interaction as explained in the sequel, but we assume that they will satisfy the relations

$$\text{tr}(B_J^\dagger B_K) = 2 \delta_{J, K}. \quad (2)$$

We also assume them to be nonsingular. Then their dimension is twice the index of nilpotency of the composites, which is the largest integer  $\nu$  such that  $(\hat{b}_J)^\nu \neq 0$ . It is obvious that a necessary condition for a composite to resemble a boson, is that its index of nilpotency be large. But this condition in general is not sufficient, and we must require also that

$$\det(\Omega B^\dagger B)^n \sim 1. \quad (3)$$

A convenient way to get the euclidean path integral from the trace of the transfer matrix is to use coherent states[11]. If we are interested in states with  $n = \bar{n} + \nu$  bosons for an arbitrary reference number  $\bar{n}$  we introduce the operator

$$\mathcal{P}_{\bar{n}} = \frac{(\Omega - \bar{n})^2}{\Omega^2} \int db^* db \langle b|b \rangle^{-1} |b\rangle \langle b| \quad (4)$$

constructed in terms of coherent states of composites

$$|b\rangle = |\exp\left(\sum_J b_J \hat{b}_J^\dagger\right)\rangle. \quad (5)$$

We would like it to be the identity in the fermion subspace of the composites. Let us see its action on composite operators. Let us first consider the case where there is only one composite with structure function satisfying the equation

$$B^\dagger B = \frac{1}{\Omega} \mathbb{1}. \quad (6)$$

In order to evaluate the matrix element  $\langle b_t | b_{t-1} \rangle$  we introduce between the bra and the ket the identity in the fermion Fock space

$$\mathcal{I} = \int dc^* dc \langle c|c \rangle^{-1} |\exp(-c^* \hat{c})\rangle \langle \exp(-c \hat{c}^\dagger)| \quad (7)$$

where the  $c^*, c$  are Grassmann variables. We thus find

$$\langle b_t | b_{t-1} \rangle = \int dc^* dc E(c^*, c, b_t^*, b_{t-1}) = \left(1 + \frac{1}{\Omega} b_t^* b_{t-1}\right)^\Omega \quad (8)$$

where

$$E(c^*, c, b^*, b) = \exp \left( -c^* c + \frac{1}{2} b^* c B c + \frac{1}{2} b c^* B^\dagger c^* \right). \quad (9)$$

Therefore the action of  $\mathcal{P}_{\bar{n}}$  on the composites

$$|\mathcal{P}_{\bar{n}}(\hat{b}^\dagger)^n\rangle = \left(1 - \frac{\nu}{\Omega - \bar{n}}\right)^{-1} \left(1 - \frac{\nu+1}{\Omega - \bar{n}}\right)^{-1} |(\hat{b}^\dagger)^n\rangle \quad (10)$$

shows that it behaves like the identity in the neighborhood of the reference state up to an error of order  $\nu/(\Omega - \bar{n})$ , namely the measure  $\langle b|b\rangle^{-1}$  is essentially uniform with respect to any reference state.

It is worth while noticing that in the limit of infinite  $\Omega$  we recover exactly the expressions valid for elementary bosons, in particular

$$\langle b_t | b_{t-1} \rangle = \left(1 + \frac{1}{\Omega} b_t^* b_{t-1}\right)^\Omega \rightarrow \exp(b_t^* b_{t-1}), \quad \Omega \rightarrow \infty. \quad (11)$$

In the general case of many composites the above equations become

$$\langle b_t | b_{t-1} \rangle = [\det(\mathbb{1} + \beta_t^* \beta_{t-1})]^\frac{1}{2}, \quad (12)$$

where  $\beta_i^* = \sum_J (b_J)_i^* B_J$ . Then using the condition 3 we find again that  $\mathcal{P}$  approximates the identity with an error of order  $1/\Omega$

$$\mathcal{P}|(\hat{b}_{I_0}^\dagger)^{n_0} \dots \hat{b}_{I_i}^\dagger)^{n_i}\rangle = |(\hat{b}_{I_0}^\dagger)^{n_0} \dots \hat{b}_{I_i}^\dagger)^{n_i} + O(1/\Omega)\rangle. \quad (13)$$

Identifying the operator  $\mathcal{P}$  with the identity in the subspace of the composites is the only approximation we will make in the derivation of the effective boson action.

### III. COMPOSITES PATH INTEGRAL

Now we are equipped to realize our program. The first step is the evaluation of the partition function  $Z_c$  restricted to fermionic composites. To this end we divide the inverse temperature in  $N_0$  intervals of spacing  $\tau$

$$\tau = \frac{1}{N_0 T} \quad (14)$$

and write

$$Z_c = \text{tr}(\mathcal{P} \exp(-H\tau))^{N_0}. \quad (15)$$

We will restrict ourselves to a Hamiltonian with interactions which can be written as a sum of pairing multipoles

$$\hat{H} = \hat{c}^\dagger h_0 \hat{c} - \sum_K g_K \frac{1}{2} \hat{c}^\dagger F_K^\dagger \hat{c}^\dagger \frac{1}{2} \hat{c} F_K \hat{c}. \quad (16)$$

The single particle term includes the single particle energy with matrix  $e$ , any single particle interaction with external fields described by the matrix  $\mathcal{M}$  and the chemical potential  $\mu$

$$h_0 = e + \mathcal{M} - \mu. \quad (17)$$

Therefore we will be able to solve the problem of fermion-boson mapping by determining the interaction of the composite bosons with external fields. We assume for the potential form factors the normalization

$$\text{tr}(F_K^\dagger F_K) = 2\Omega. \quad (18)$$

For the following manipulations we need the Hamiltonian in antinormal form

$$\hat{H} = H_0 - \hat{c} h^T \hat{c}^\dagger - \sum_K g_K \frac{1}{2} \hat{c} F_K \hat{c} \frac{1}{2} \hat{c}^\dagger F_K^\dagger \hat{c}^\dagger \quad (19)$$

where the upper script  $T$  means "transposed" and

$$h = h_0 - \sum_K g_K F_K^\dagger F_K, \quad H_0 = \frac{1}{2} \text{tr}(h + h_0). \quad (20)$$

Now we must evaluate the matrix element  $\langle b_t | \exp(-\tau \hat{H}) | b_{t-1} \rangle$ . To this end we expand to first order in  $\tau$  (which does not give any error in the final  $\tau \rightarrow 0$  limit) and insert the operator  $\mathcal{P}$  between annihilation and creation operators

$$\begin{aligned} \langle b_t | \exp(-\tau \hat{H}) | b_{t-1} \rangle &= \exp(-H_0 \tau) \langle b_t | \mathcal{P} - \hat{c} h^T \tau \mathcal{P} \hat{c}^\dagger \\ &\times \sum_K g_K \tau \frac{1}{2} \hat{c} F_K \hat{c} \mathcal{P} \frac{1}{2} \hat{c}^\dagger F_K^\dagger \hat{c}^\dagger | b_{t-1} \rangle. \end{aligned} \quad (21)$$

Using the identity in the fermion Fock space we find

$$\begin{aligned} \langle b_t | \exp(-\tau \hat{H}) | b_{t-1} \rangle^{-1} &= \int dc^* dc E(c^*, c, b_t^*, b_{t-1}) \\ &\times \exp(-H_0 \tau - c^* h \tau c) \exp \left( \sum_K g_K \tau \frac{1}{2} c F_K c \frac{1}{2} c^* F_K^\dagger c^* \right) \end{aligned} \quad (22)$$

where the function  $E(c^*, c, b^*, b)$  is defined in (9). By means of the Hubbard-Stratonovich transformation we can make the exponents quadratic in the Grassmann variables and evaluate the Berezin integral

$$\begin{aligned} \langle b_t | \exp(-\tau \hat{H}) | b_{t-1} \rangle &= \det R \exp(-H_0 \tau) \int \prod_K da_K^* da_K \\ &\times \exp(-a^* \cdot a) \exp \left\{ \frac{1}{2} \text{tr} \ln \left[ \mathbb{1} + \left( \beta_t^* + \sum_{K_1} \sqrt{g_{K_1} \tau} a_{K_1}^* F_{K_1} \right) \right. \right. \\ &\times R^{-1} \left. \left( \beta_{t-1} + \sum_{K_2} \sqrt{g_{K_2} \tau} a_{K_2} (F_{K_2})^\dagger \right) (R^T)^{-1} \right] \right\}, \end{aligned} \quad (23)$$

where  $R = \mathbb{1} + h \tau$ . Setting  $\Gamma_t = (\mathbb{1} + \beta_t^* \beta_{t-1})^{-1}$  and performing the integral over the auxiliary fields  $a_K^*, a_K$  we get the final expression of the euclidean action

$$\begin{aligned} S(b^*, b) &= \tau \sum_t \left\{ H_0 - \text{tr} h + \frac{1}{2\tau} \text{tr} [\ln(\mathbb{1} + \beta_t^* \beta_t) + \ln \Gamma_t] \right. \\ &- \frac{1}{4} \sum_K g_K \left[ \text{tr}(\Gamma_t \beta_t^* F_K^\dagger) \text{tr}(\Gamma_t F_K \beta_{t-1}) + 2 \text{tr}(\Gamma_t F_K^\dagger F_K) \right. \\ &\left. \left. - \text{tr}[\Gamma_t \beta_t^* F_K^\dagger, \Gamma_t F_K \beta_{t-1}]_+ \right] + \frac{1}{2} \text{tr} [\Gamma_t \beta_t^* (\beta_{t-1} h^T + h \beta_{t-1})] \right\} \end{aligned} \quad (24)$$

where  $[\dots]_+$  is an anticommutator. This action differs from that of elementary bosons because

i) the time derivative terms are non canonical. Indeed expanding the logarithms we get

$$\text{tr} [\ln(\mathbb{1} + \beta_t^* \beta_t) + \ln \Gamma_t] = \frac{1}{2} \left\{ \beta_t^* \nabla_t \beta_t - \frac{1}{2} [(\beta_t^*)^2 \nabla_t (\beta_t^*)^2] + \dots \right\}, \quad (25)$$

where  $\nabla_t f = \frac{1}{\tau} (f_{t+1} - f_t)$ . The first term is the canonical one, while the others contain the derivative of powers of the boson variables. The canonical form of the first term is due to the normalization of Eq.(2) of the structure functions, otherwise  $\beta_t$  and  $\beta_{t-1}$  would not have the same coefficient. Note the difference of the noncanonical terms with respect to the chiral expansions, where there are powers of derivatives, rather than derivatives of powers.

ii) the coupling of the chemical potential (which appears in  $h$ ) is also noncanonical. Indeed expanding  $\Gamma_t$  we get  $\mu \text{tr} (\beta_t^* \beta_{t-1} - \beta_t^* \beta_t \beta_t^* \beta_{t-1} + \dots)$ , and only the first term is canonical

iii) the function  $\Gamma$  becomes singular when the number of bosons is of order  $\Omega$ , as it will become clear in the sequel. This reflects the Pauli principle.

We remind the reader that the only approximation done concerns the operator  $\mathcal{P}$ . Therefore these are to be regarded as true features of compositeness.

The bosonization of the system we considered has thus been accomplished. In particular the fermionic interactions with external fields can be expressed in terms of the bosonic terms which involve the matrix  $\mathcal{M}$  (appearing in  $h$ ). The dynamical problem of the interacting (composite) bosons can be solved within the path integral formalism. This includes the new interesting possibility of a numerical simulation of the partition function which could now be performed with bosonic variables avoiding the sign problem.

Part of the dynamical problem is the determination of the structure matrices  $B_J$ . This can be done by expressing the energies in terms of them and applying a variational principle which gives rise to an eigenvalue equation.

#### IV. THE ACTION IN A SINGLE $j$ -SHELL

In this paper we restrict ourselves to a system of nucleons of in a single  $j$ -shell. Then we identify the quantum number  $K$  with the boson angular momentum,  $K = (I_K, M_K)$ , so that the form factors of the potential are proportional to Clebsch-Gordan coefficients

$$(F_{IM})_{m_1, m_2} = \sqrt{2\Omega} C_{jm_1 j m_2}^{IM}, \quad \Omega = j + \frac{1}{2}. \quad (26)$$

In such a case the structure matrices are completely determined by the angular momentum of the composites and the normalization conditions (2)  $B_J = \Omega^{-\frac{1}{2}} F_J$ . The points i) and ii) following Eq. (24) are the only difficulties in the derivation of the Hamiltonian which could be otherwise read from the action. We can overcome them by performing an expansion in inverse powers of  $\Omega$ . We will retain only the first order corrections, which are of order  $\Omega^0$ , with the exception of the coupling with external fields where they are of order  $\Omega^{-1}$ . In this approximation the first difficulty is overcome because noncanonical time derivatives are of order  $1/\Omega$  and the second one because the only noncanonical coupling of the chemical potential of order  $\Omega^0$  comes from the only term of the chemical potential of order  $\Omega$ , which can be shown to be  $\mu \sim -\frac{1}{2} g_0 \Omega$ , independent of the number of bosons.

The resulting action is

$$\begin{aligned} S(b^*, b) = & \sum_t \left\{ \sum_{K_1 K_2} b_{K_1}^* [(\nabla_t - 2\mu) + \omega]_{K_1 K_2} b_{K_2} \right. \\ & \left. + \sum_{I_1 I_2 I_3 I_4} \sum_{IM} W_{I_1 I_2 I_3 I_4}^I (b_{I_1}^* b_{I_2}^*)_{IM} (b_{I_3} b_{I_4})_{IM} \right\} \end{aligned} \quad (27)$$

where all the  $b^*$ 's and all the  $b$ 's are at time  $t, t-1$  respectively, and

$$\begin{aligned} \omega_{K_1 K_2} &= \frac{1}{\Omega} \text{tr} \left( F_{K_1} F_{K_2}^\dagger e \right) - g_{I_1} \Omega \delta_{K_1 K_2} \\ W_{I_1 I_2 I_3 I_4}^I &= \left( -2g_0 + \sum_{i=1}^4 g_{I_i} \right) \Pi_{i=1}^4 [(2I_i + 1)]^{1/2} \Omega \begin{Bmatrix} j & j & I_1 \\ j & j & I_2 \\ I_3 & I_4 & I \end{Bmatrix} \\ (b_{I_3} b_{I_4})_{IM} &= \sum_{M_3, M_4} C_{I_3, M_3, I_4, M_4}^{I, M} b_{I_3 M_3} b_{I_4 M_4}. \end{aligned} \quad (28)$$

Notice the factor 2 in front of the chemical potential due to the fact that the composites have fermion number 2.

#### V. THE HAMILTONIAN

The Hamiltonian is obtained[11] by omitting the time derivative and chemical potential terms, and replacing the variables  $b^*, b$  by corresponding creation-annihilation operators  $\hat{a}^\dagger, \hat{a}$ , satisfying canonical commutation relations

$$\begin{aligned} \hat{H} = & \sum_{I_1 M_1 I_2 M_2} \omega_{I_1 M_1 I_2 M_2} \hat{a}_{I_1 M_1}^\dagger \hat{a}_{I_2 M_2} + \sum_{I_1 I_2 I_3 I_4} \sum_{IM} \\ & \left\{ W_{I_1 I_2 I_3 I_4}^I \left( \hat{a}_{I_1}^\dagger \hat{a}_{I_2}^\dagger \right)_{IM} (\hat{a}_{I_3} \hat{a}_{I_4})_{IM} \right\}. \end{aligned} \quad (29)$$

It is easy to check that, due to the symmetries of the 9j symbols, it is hermitian.

From the interaction with external fields we get the fermion-boson mapping of other operators

$$\begin{aligned}
\hat{c}^\dagger \mathcal{M} \hat{c} &\rightarrow \sum_{I_1 M_1 I_2 M_2} \frac{2}{\Omega} \text{tr} \left( F_{I_1 M_1} \mathcal{M} F_{I_2 M_2}^\dagger \right) \hat{a}_{I_1 M_1}^\dagger \hat{a}_{I_2 M_2} \\
&+ \sum_{\text{all } I, M} \left( \frac{2}{\Omega} \right)^2 \text{tr} \left( F_{I_1 M_1} \mathcal{M} F_{I_4 M_4}^\dagger F_{I_2 M_2} F_{I_3 M_3}^\dagger \right) \\
&\times \hat{a}_{I_1 M_1}^\dagger \hat{a}_{I_2 M_2}^\dagger \hat{a}_{I_3 M_3} \hat{a}_{I_4 M_4}.
\end{aligned} \tag{30}$$

We remind the reader that the above Hamiltonian has been derived under the condition  $n \ll \Omega$  in a single subshell. Therefore if we further assume  $e_{m_1 m_2} = \bar{e} \delta_{m_1 m_2}$ , the single boson energy matrix is diagonal  $\omega_{I_1 I_2} = (2\bar{e} - g_{I_1} \Omega) \delta_{I_1 I_2}$ . But the bosonic interactions couple all the bosons with angular momenta for which the  $9j$  symbols do not vanish, even if the corresponding potentials do vanish.

## VI. SUMMARY

We have developed a general approach to the problem of bosonization where we introduce fermionic composites without any preliminary mapping of the fermion model space into a bosonic one. Restricting the trace in the partition function of the system to the composites we get the euclidean action of the effective bosons in closed form. The only approximation made concerns the identity operator in the space of the composites.

It is perhaps worth while to spend a few words about the nature of this approximation. Indeed it might appear that two are the approximations involved. The first one is the restriction of the partition function to composites. This is the fundamental physical assumption of Boson Dominance. Then we replace the identity in the composite subspace by the operator  $\mathcal{P}$ , which seems a further approximation. But  $\mathcal{P}$  differs appreciably from the identity only for states with many bosons, states which cannot resemble elementary bosons because of the Pauli principle. We therefore deem that the two approximations are essentially one and the same.

The nuclear dynamics can be studied by the methods of path integrals, including numerical simulations which now are not affected by the sign problem.

To derive the Hamiltonian of the IBM we must make recourse to an expansion in the inverse of the index of nilpotency of the composites. In the present work we restricted ourselves to a nucleon model space of a single  $j$ -shell and to a number of bosons much smaller than the index of nilpotency. Both limitations can easily be removed. Concerning the second one, some care must be exercised to respect particle number conservation, as done for instance in [7]. The first one requires a parametrization of the structure functions according to

$$(B_{J,M})_{m_1, m_2} = \sum_{j_1 j_2} p_{J j_1 j_2} C_{j_1 m_1 j_2 m_2}^{JM}. \tag{31}$$

Now the energies of the bosons are functions of the parameters  $p$ . A variational principle applied to these energies generates an equation for these parameters. The solution to this equation can in general be found only numerically, but the Hamiltonian and the other operators retain their analytic expressions.

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